How Electromagnetic Effects Affect the Stability of Plasmas in Tokamaks

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1. Introduction

Fusion is a process of combining light elements to form heavier elements with an associated release of energy. Isotopes of hydrogen can be fused to form helium. For the elements to have enough energy to fuse they need to be at a temperature of 150 million °C. To contain plasma at such high temperatures it is suspended in a magnetic field created by a system of magnets in a tokamak (Figure 1). Owing to to turbulence in the plasma, confinement can be lost resulting in hot plasma making contact with the walls of the reactor and causing damage. This is an issue which must be resolved if we want fusion reactors that can generate power. These turbulent plasma ejections occur in the scape off layer (SOL), a thin layer of plasma making up the surface of the confined torus volume of plasma (Figure 2).

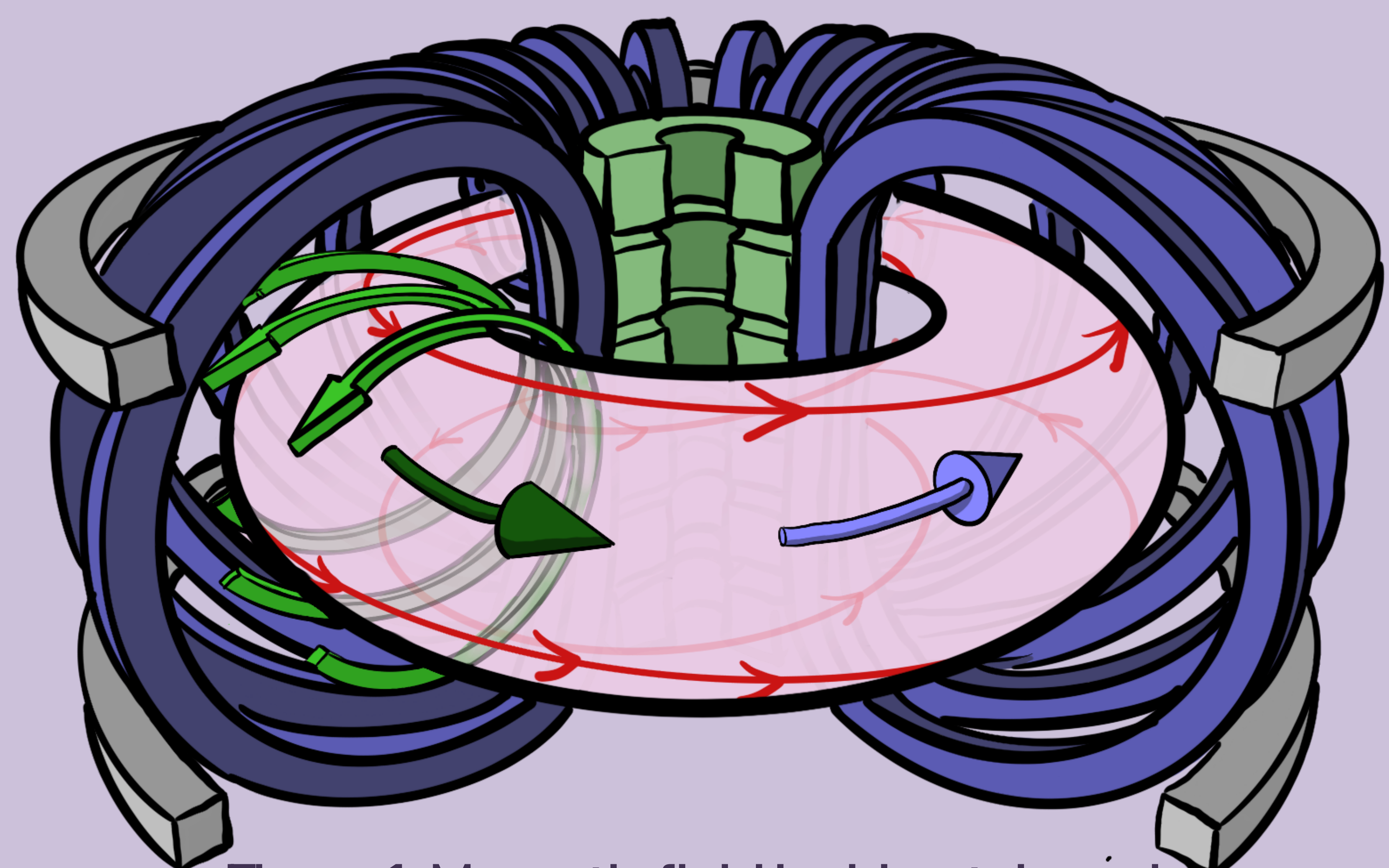

Figure 1: Magnetic field inside a tokamak

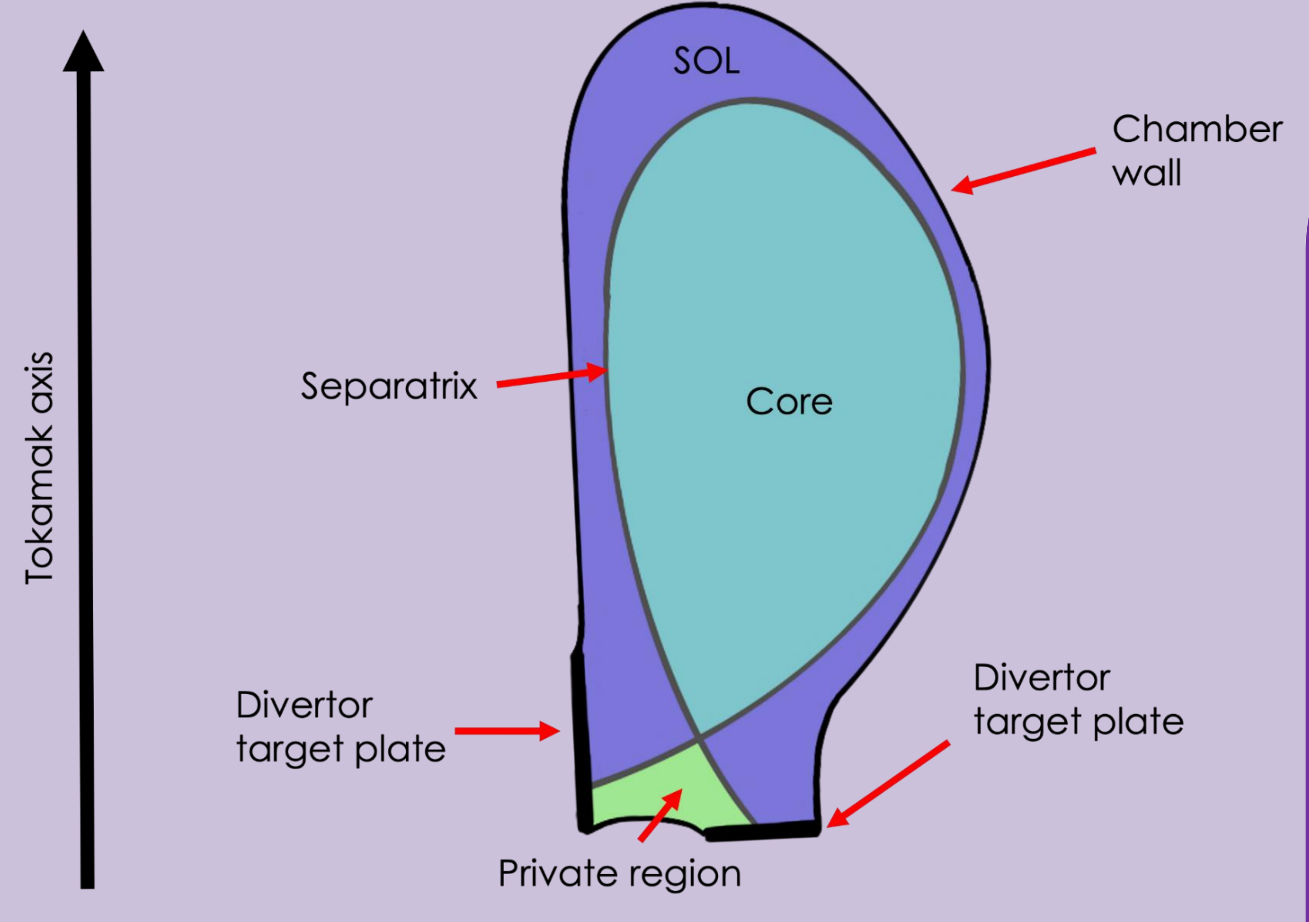

Figure 2: Main regions of the plasma including the scrape off layer (SOL)

2. Governing Equations

As shown in Figure 3, the plasma system can be regarded as analogous to Rayleigh–Bénard convection (RBC), where we consider a density gradient across the SOL. The effective gravity term "g" acts in the radial direction and models the curvature and gradient of the magnetic field. The dynamics of the plasma in our model are governed by the vorticity equation, particle density equation and Ohm's law given by,

$$\frac{m_i n}{B}\left(\frac{\partial}{\partial t}+\boldsymbol{v}_E\cdot\boldsymbol{\nabla}\right)\omega=-\frac{1}{\mu_0}\nabla_\parallel\nabla_\perp^2 A_\parallel-\frac{eg}{\Omega_i}\frac{\partial n}{\partial y}+\frac{nm_i\nu}{B}\nabla_\perp^2\omega, \qquad (1)$$

$$\frac{\partial n}{\partial t}+\boldsymbol{v}_E\cdot\boldsymbol{\nabla}n+\frac{1}{\mu_0 e}\nabla_\parallel\nabla_\perp^2 A_\parallel=\frac{gn}{Bc_s^2}\frac{\partial\varphi}{\partial y}-\frac{g}{\Omega_i}\frac{\partial n}{\partial y}+D\nabla_\perp^2 n, \qquad (2)$$

$$\frac{\partial A_\parallel}{\partial t}=\frac{1}{\mu_0\sigma_\parallel}\nabla_\perp^2 A_\parallel+\frac{T_e}{en}\nabla_\parallel n-\nabla_\parallel\varphi, \qquad (3)$$

where m_i is the mass of the ions, n is the plasma particle density, $\boldsymbol{v}_E$ is the velocity with which particles drift from confinement due to the Lorentz force, c_s is the plasma sound speed, Ω_i is the ion gyrofrequency, D is the collisional diffusion constant, $A_\parallel$ is the parallel component of the magnetic potential, φ is the electric potential, $\sigma_\parallel$ is the parallel component of the conductivity, T_e is the electron temperature, μ_0 is permeability of free space and e is the elementary charge.

3. Linear Stability Analysis

By introducing a small perturbation into the basic state (tilde variables) of equations (1-3) and retaining only linear terms, we obtain (4-6). Here we have non-dimensionalised the equations and introduced the following non-dimensional variables: the effective Rayleigh number, $Ra^* = gh^3/D\nu_i$; the Prandtl number, $Pr = \nu_i/D$; the Chandrasekhar number, $Q = B^2h^2\sigma_\parallel/n_0\nu_i m_i$; the ratio of the gyrofrequency to the time scale of diffusion, $\Omega = \Omega_i h^2/D$, the scaled ratio of the SOL widths to the tokamak radius of curvature, $\lambda = 2h/R_c$ and the ratio of the magnetic diffusivity to collisional diffusion, $\zeta = \eta/D$.

$$\tilde{n}(x)\frac{\partial}{\partial t}\nabla_\perp^2\varphi=-QPr\zeta\nabla_\parallel\nabla_\perp^2 A_\parallel-Ra^*Pr\frac{\partial n}{\partial y}+\tilde{n}(x)Pr\nabla_\perp^2\nabla_\perp^2\varphi \qquad (4)$$

$$\frac{\partial n}{\partial t}+\frac{QPr}{\Omega}\zeta\nabla_\parallel\nabla_\perp^2 A_\parallel=-\Delta N(x)\frac{\partial\varphi}{\partial y}-\frac{Ra^*Pr}{\Omega}\frac{\partial n}{\partial y}+\nabla_\perp^2 n \qquad (5)$$

$$\tilde{n}(x)\frac{\partial A_\parallel}{\partial t}=\zeta\tilde{n}(x)\nabla_\perp^2\mathrm{A}_\parallel+\frac{\mathrm{Ra^*Pr}}{\lambda\Omega}\nabla_\parallel n-\tilde{n}(x)\nabla_\parallel\varphi \qquad (6)$$

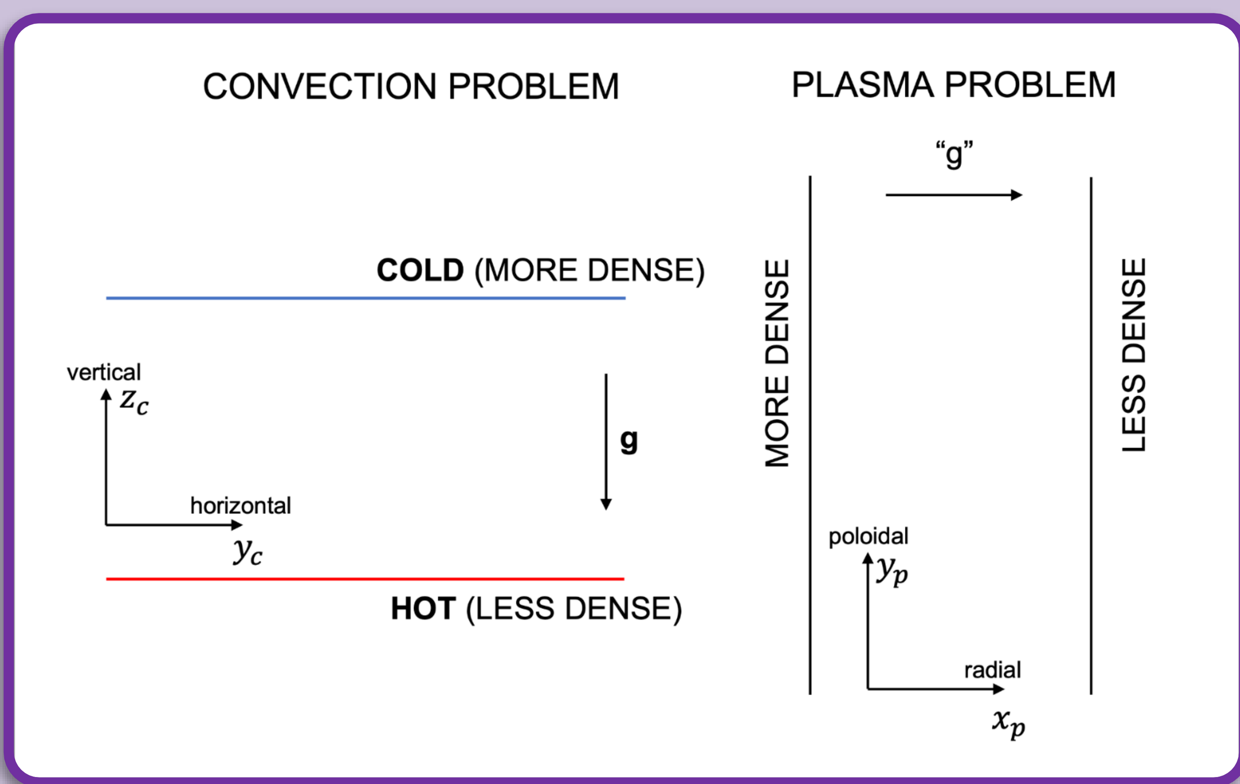

Figure 3: RBC plasma analogy

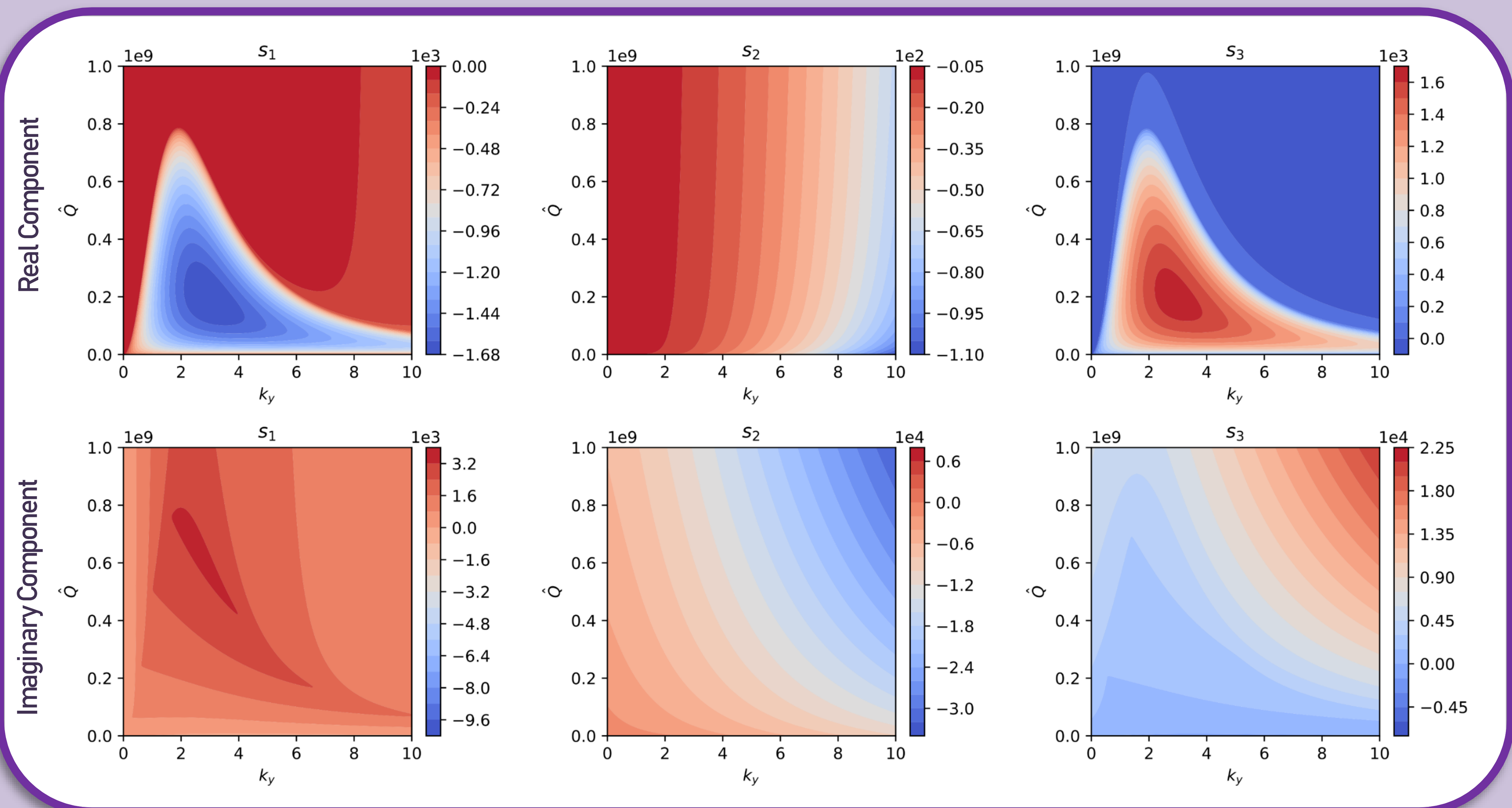

Figure 4: Real and imaginary parts of the roots s of (7) for a range of $\hat{Q}$ and k_y values.

4. Simplified Analytical Model

To analyse the stability of (4-6) we start with a small density gradient approximation, $\tilde{n}(x) = 1, d\tilde{n}/dx = \text{constant}$, thus neglecting the x dependence. We postulate plane wave solutions of the form, $\varphi = \hat{\varphi}\sin(m\pi x)\exp(st + ik_y y + ik_z z)$. Where m is an integer, k_y and k_z are the wavenumbers in the poloidal and toroidal directions and s is the complex growth rate. If $\mathrm{Re}(s)$ is negative this represents a stable state, and if positive an unstable state. $\mathrm{Re}(s) = 0$ denotes a marginal state on the onset of instability. Solving (4-6) with our approximation, where we also have $\hat{Q} = Qk_z^2$, yields,

$$(s+\zeta)\left((-s\Delta_k - Pr\Delta_k^2)\left(s+\frac{Ra^*Pr}{\Omega}ik_y+\Delta_k\right)+Ra^*Prk_y^2\Delta N\right)$$
$$=\frac{\hat{Q}Pr}{\Omega}\zeta\Delta_k\left(\frac{Ra^*Pr}{\lambda\Omega}\left(s\Delta_k+Pr\Delta_k^2-\Delta_k^2-\Delta Nik_y\Omega-2ik_y\lambda\Omega\right)-(s+\Delta_k)\Omega\right). \qquad (7)$$

5. Preliminary Results and Future Work

In Figure 4 we have plotted $\mathrm{Re}(s)$ and $\mathrm{Im}(s)$ of (7) for a range of $\hat{Q}$ and k_y values with $Ra^* = 10^8, Pr = 1,\ \Omega = 10^5,\ \lambda = 0.04$, $\zeta = 10^{-2}, \Delta N = 0.0241$. The third root has a clear region of instability. We will continue this investigation in the linear regime, first by a thorough investigation of parameter space for the simplified model, then by dropping the small gradient approximation. Our ultimate goal will be to consider the nonlinear evolution governed by (1-3).

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